Core 1

Key skills

This booklet contains the assumed knowledge for AS Mathematics. If you are unsure of any of the following topics, help can be found online at:

- **Hegarty Maths**
- **Corbett Maths**
- **BBC bitesize**
- **You tube**

In the first week of Year 12 you will sit a baseline assessment based on the questions in this pack. This may be used to determine your class and gauge your understanding of the fundamental principles of algebra.

NO CALCULATORS ALLOWED

Surds

1. Evaluate

a) $\sqrt[3]{8}$ b) $\sqrt[4]{81}$ c) $\sqrt{7} \times \sqrt{7}$ d) $(3\sqrt{3})^2$

2. Simplify

a) $\sqrt{18}$ b) $\sqrt{50}$ c) $\sqrt{27}$ d) $\sqrt{99}$

3. Simplify

a) $\sqrt{18} + \sqrt{50}$ b) $\sqrt{48} - \sqrt{27}$

4. Expand and Simplify

a) $\sqrt{3}(2 + \sqrt{3})$ b) $(\sqrt{5} + 1)(2\sqrt{5} + 3)$

5. Rationalise the denominator

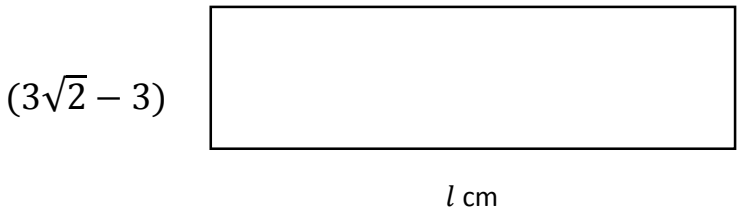
a) $\frac{1}{\sqrt{5}}$ b) $\frac{3\sqrt{2}}{\sqrt{3}}$ c) $\frac{4\sqrt{20}}{3\sqrt{18}}$

6. Rationalise the denominator

a) $\frac{1}{\sqrt{2}+1}$ b) $\frac{3}{2+\sqrt{3}}$ c) $\frac{\sqrt{2}}{\sqrt{2}+\sqrt{6}}$ d) $\frac{3-\sqrt{2}}{4+3\sqrt{2}}$

7. (Exam Style) The rectangle below measuring $(3\sqrt{2}-3)$ cm and l cm.

Given that the area is 6cm. Find the value of l in the form $a\sqrt{b}+c$, where a, b, c are integers.



Indices

1. Simplify

a) $2p^2 \times p^3$ b) $\frac{18rs^2}{6r^3s}$ c) $\frac{2x^2y^3}{8x^5y}$

2. Evaluate

a) $(3)^{-2}$ b) $9^{\frac{1}{2}}$ c) $8^{-\frac{1}{3}}$ d) $(\frac{4}{9})^{-\frac{1}{2}}$

3. Simplify (leaving the indices as fractions)

a) $y^3 \times y^{\frac{1}{2}}$ b) $\frac{d^2}{d^{-\frac{1}{3}}}$ c) $f^{\frac{3}{2}} \times f^{-\frac{1}{5}}$

4. Express each of the following in the form ax^b

a) $\frac{4}{\sqrt{x}}$ b) $\frac{3}{4x^3}$ c) $\frac{1}{\sqrt{9x^3}}$ d) $\frac{2}{5\sqrt[3]{x}}$

5. Write each of the following in the form 2^k.

a) 8^2 b) $\left(\frac{1}{2}\right)^{\frac{1}{3}}$ c) $8^{\frac{2}{3}}$ d) $\left(\frac{1}{32}\right)^{-\frac{1}{5}}$

6. Expand and simplify.

a) $x^{-\frac{3}{2}}(5x^2 + x^{\frac{7}{2}})$ b) $(x^2 - x^{\frac{3}{2}})(x - x^{\frac{1}{2}})$

7. Simplify.

a) $\frac{x^3+2x}{x}$ b) $\frac{x^{\frac{3}{2}}-3x}{x^{\frac{1}{2}}}$ c) $\frac{2t^3-4t}{t^{\frac{3}{2}}-2t^{-\frac{1}{2}}}$

8. Exam Style

Given that $y = 2^x$, express each of the following in terms of y.

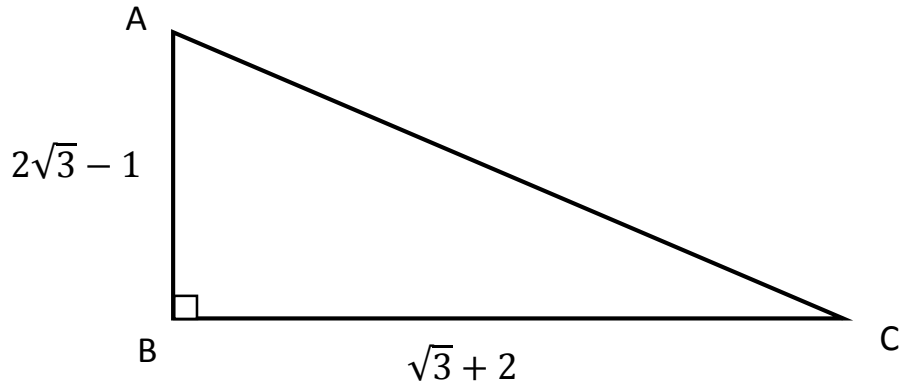
a) 2^{x+2} b) 4^x

c) Hence find the exact value of $4^x - 2^{x+2} = 0$

9. Exam style

Given that the point with coordinates $(1 + \sqrt{3}, 5\sqrt{3})$ lies on the curve with the equation $y = x^2 + px + q$. Find the values of p and q.

10. Exam Style



a) Find the exact area of the triangle in its simplest form.
b) Show that $AC = 2\sqrt{5}$.
c) Show that $\tan(ACB) = 5\sqrt{3} - 8$

Quadratics

1. Factorise

   a) $x^2 + 4x + 3$ b) $x^2 + 2x - 8$ c) $x^2 - 13x + 40$

2. Factorise

   a) $2x^2 + 3x + 1$ b) $9x^2 - 6x + 1$ c) $4x^2 - 25$

3. By factorising solve the following equations

   a) $x^2 + 6x + 8 = 0$ b) $2x^2 - 3x + 1 = 0$ c) $6x^2 + 10 = 19x$

4. Factorise FULLY.
   a) $2x^2 - 10x + 12$ b) $3x^3 + 21x^2 + 18x$ c) $x^4 - 1$

5. By completing the square, write the following in the form $(x + a)^2 + b$.
   a) $x^2 + 2x + 4$ b) $x^2 - 8x - 5$ c) $x^2 - 7x - 2$

6. By completing the square, write the following in the form $a(x + b)^2 + c$.
   a) $2x^2 - 8x - 7$ b) $2x^2 + 4x + 3$ c) $3x^2 - 9x + 5$

Example (Sketching)

Take $y = x^2 - 8x + 15$

When $y = 0$
Factorise and solve $x^2 - 8x + 15 = 0$

- $(x - 3)(x - 5) = 0$
- Therefore $x = 3$ and $x = 5$

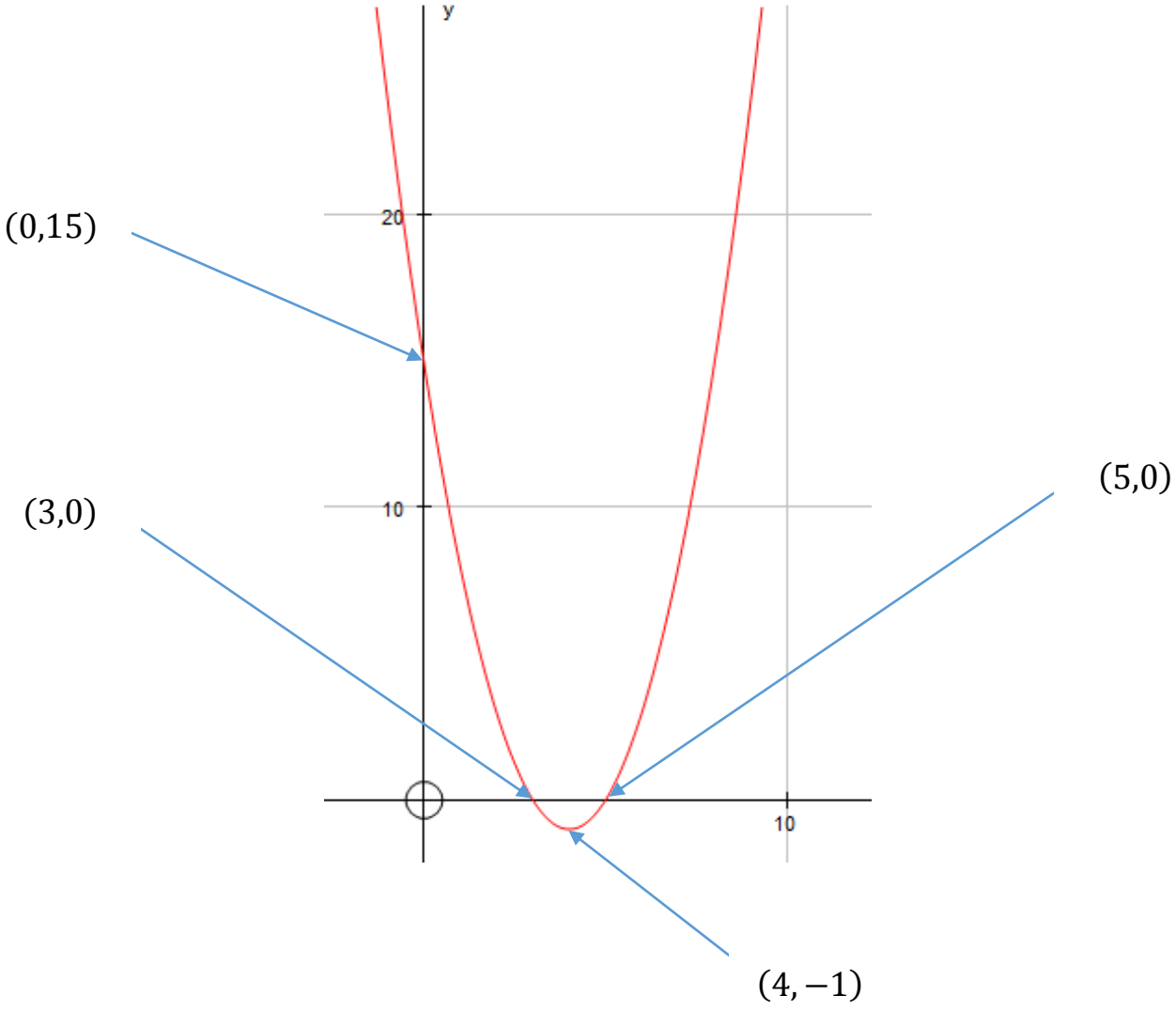
When $x = 0$

- $y = 0^2 - 8(0) + 15$
- Therefore $y = 15$

Then complete the square

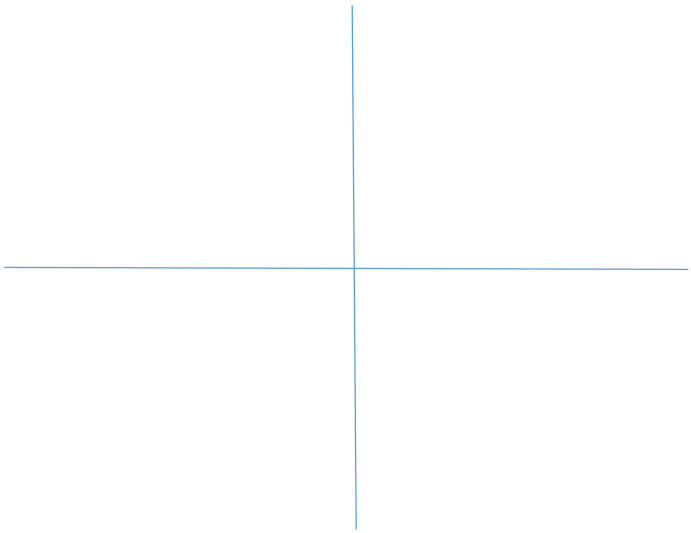
- $y = (x - 4)^2 - 1$
- Therefore the minimum point is of the parabola is $(4, -1)$.

Therefore sketching the curve $y = x^2 - 8x + 15$

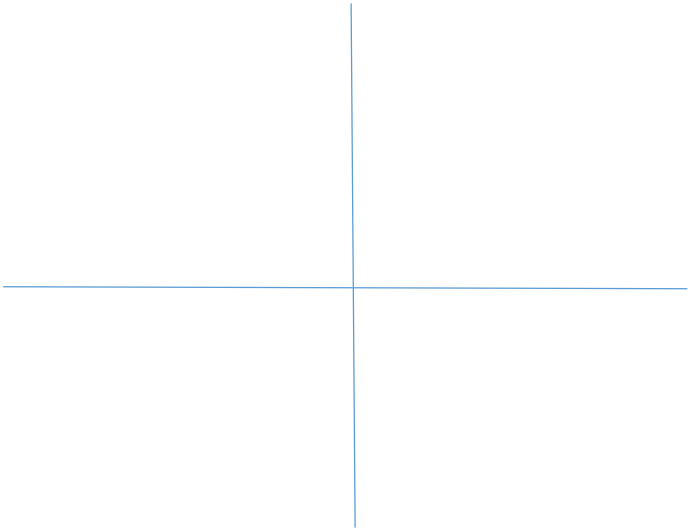


Sketch the curve for each of the following equations showing the minimum point and where it crosses the x and y axis.

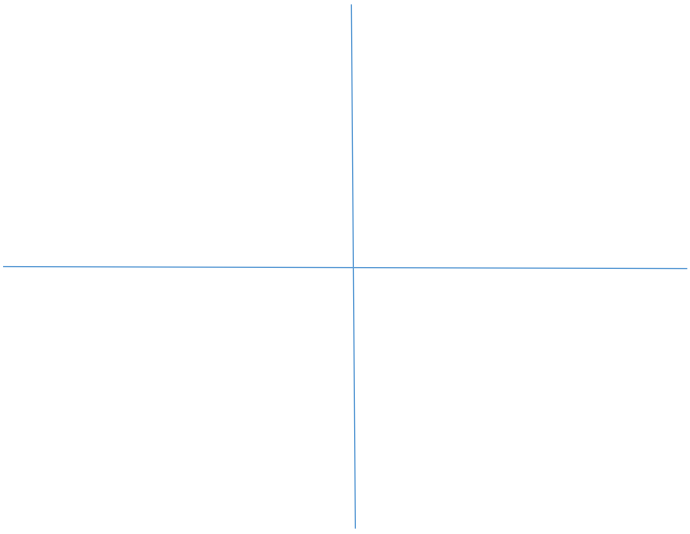
1. $y = x^2 - 4x + 3$



2. $y = x^2 + 2x + 1$



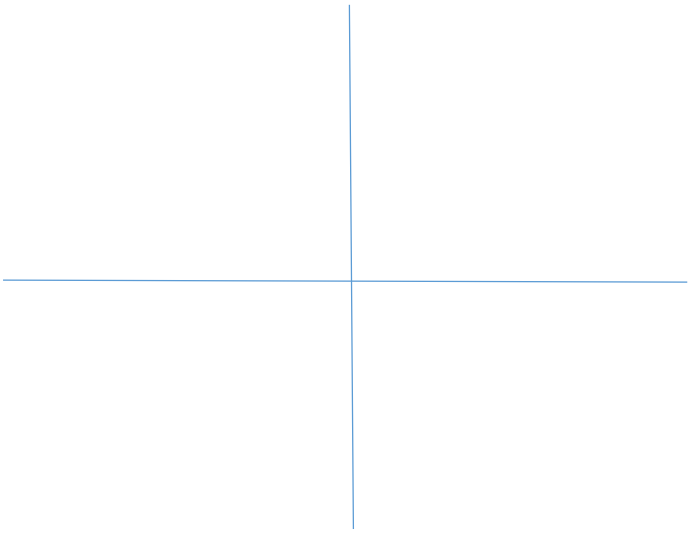
3. $y = -x^2 - 4x - 7$



Exam Style Question

a) Express $x^2 + 4\sqrt{2}x + 5$ in the form $a(x + b)^2 + c$

b) Sketch the equation of the graph $y = x^2 + 4\sqrt{2} + 5$



Simultaneous Equations

Use an appropriate method to solve the following simultaneous equations.

1. $y = 3x$
   $y = 2x + 1$

2. $3x + 3y + 4 = 0$
   $5x - 2y - 5 = 0$

3. $y = 4x + 11$
   $y = x^2 + 3x - 1$

Answers

Surds

1 a) 2 b) 3 c) 7 d) 27

2 a) $3\sqrt{2}$ b) $5\sqrt{2}$ c) $3\sqrt{3}$ d) $3\sqrt{11}$

3 a) $8\sqrt{2}$ b) $\sqrt{3}$

4 a) $2\sqrt{3}+3$ b) $13+5\sqrt{5}$

5 a) $\frac{\sqrt{5}}{5}$ b) $\sqrt{6}$ c) $\frac{4\sqrt{10}}{9}$

6 a) $\sqrt{2}-1$ b) $6-3\sqrt{3}$ c) $\frac{\sqrt{3}-1}{2}$ d) $\frac{13\sqrt{2}-18}{2}$

7 $\sqrt{2}+2$

Indices

1 a) $2p^5$ b) $\frac{3s}{r^2}$ c) $\frac{y^2}{4x^3}$

2 a) $\frac{1}{9}$ b) 3 c) $\frac{1}{2}$ d) $\frac{3}{2}$

3 a) $y^{\frac{7}{2}}$ b) $d^{\frac{7}{3}}$ c) $f^{\frac{13}{10}}$

4 a) $4x^{-\frac{1}{2}}$ b) $\frac{3}{4}y^{-3}$ c) $\frac{1}{3}y^{-\frac{3}{2}}$ d) $\frac{2}{5}x^{-\frac{1}{3}}$

5 a) 2^6 b) $2^{-\frac{1}{3}}$ c) 2^2 d) 2^1

6 a) $5x^{\frac{1}{2}}+x^2$ b) $x^3+x^2-x^{\frac{5}{2}}$

7 a) x^2+2 b) $x-3x^{\frac{1}{2}}$ c) $\frac{t^2-4}{t^{\frac{1}{2}}-2t^{-\frac{3}{2}}}$

8 a) $4y$ b) y^2 c) $y=0$ and $y=2$

9 $p=1\ and\ q=9$

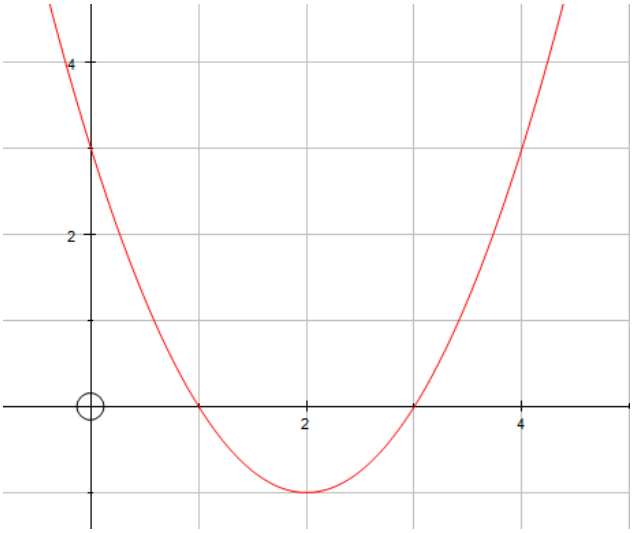
10 a) $2+\frac{3}{2}\sqrt{3}$ b) Proof c) Proof

Quadratics

1	a)	$(x+3)(x+1)$	b)	$(x+4)(x-2)$	c)	$(x-8)(x-5)$
2	a)	$(2x+1)(x+1)$	b)	$(3x-1)(3x-1)$	c)	$(2x+5)(2x-5)$
3	a)	$x=-2, x=-4$	b)	$x=\frac{1}{2}, x=1$	c)	$x=\frac{2}{3}, x=\frac{5}{2}$
4	a)	$2(x-2)(x-3)$	b)	$3x(x+6)(x+1)$	c)	$(x^2+1)(x+1)(x-1)$
5	a)	$(x+1)^2+3$	b)	$(x-4)^2-21$	c)	$\left(x-\frac{7}{2}\right)^2-\frac{57}{4}$
6	a)	$2(x-2)^2-\frac{23}{2}$	b)	$2(x+1)^2+1$	c)	$3\left(x-\frac{3}{2}\right)^2-\frac{7}{4}$

Sketching

1



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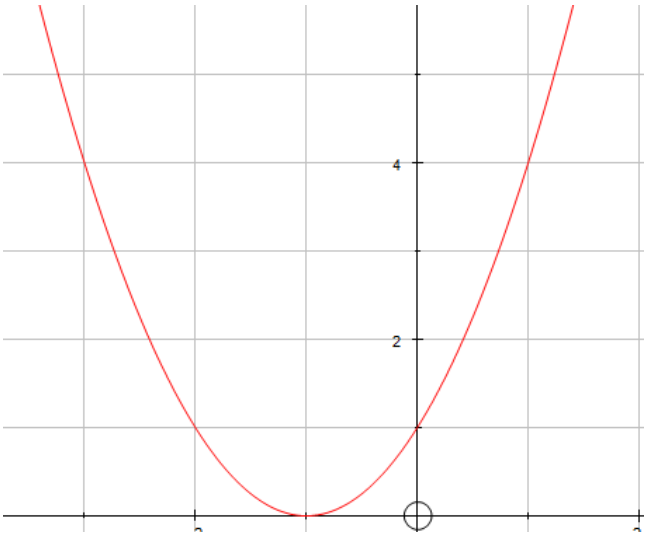

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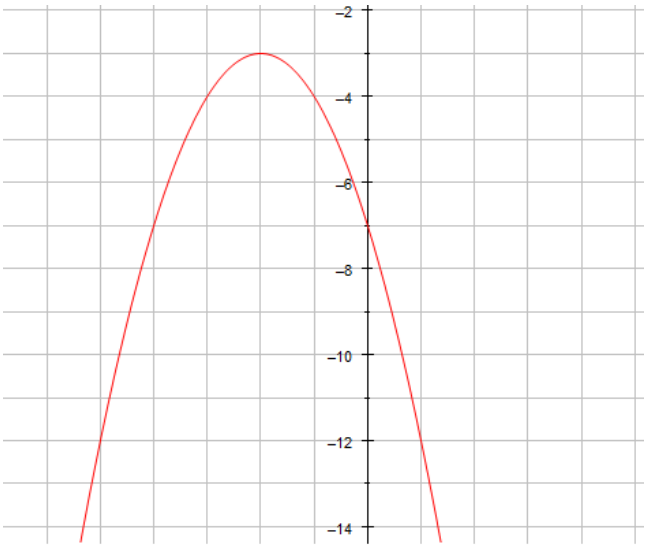
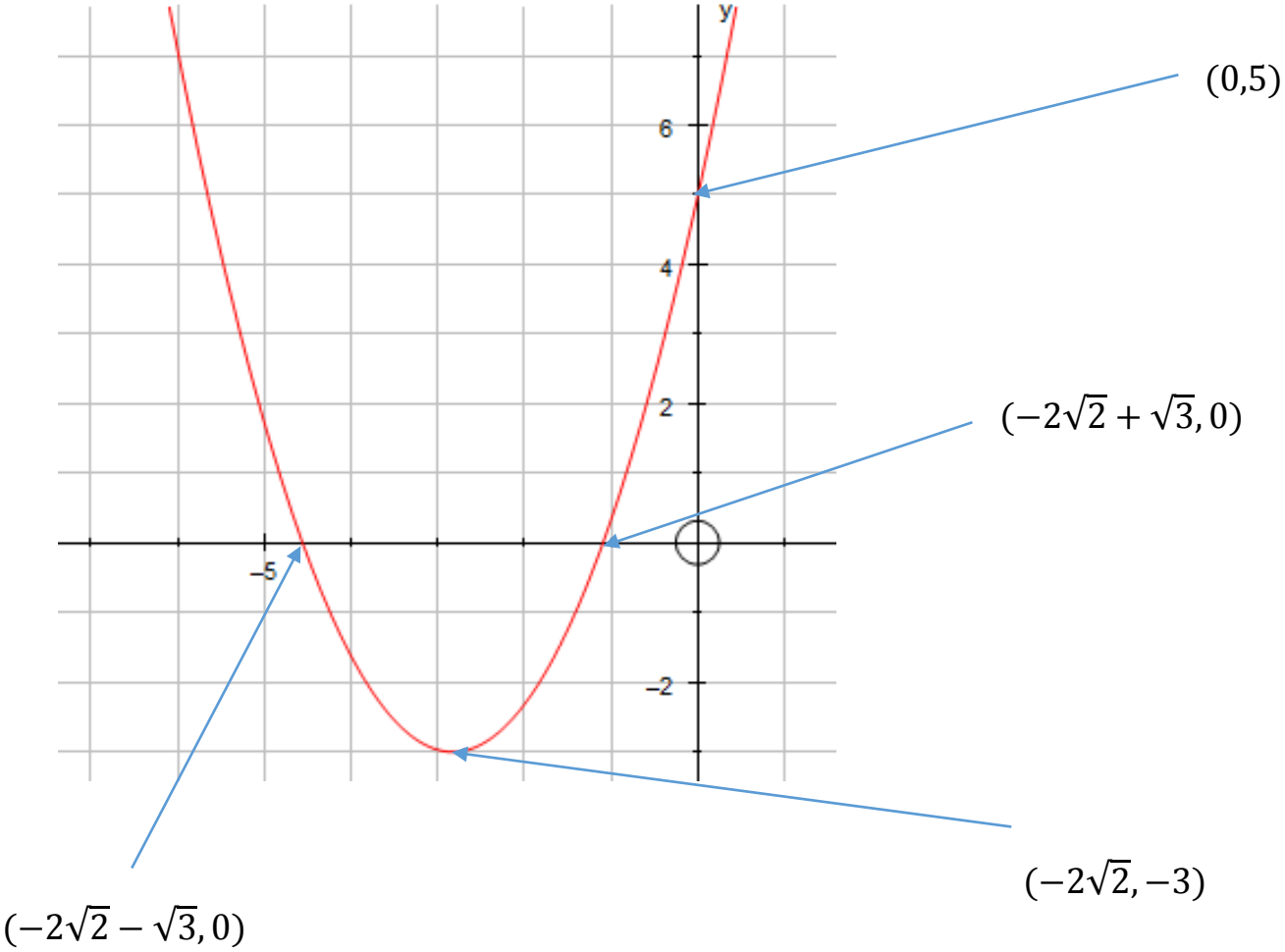


Exam Question

a) $\left(x + 2\sqrt{2}\right)^2 - 3$

b)



Simultaneous Equations

1 $x = 1 \; and \; y = 3$

2 $x = \frac{1}{3} \; and \; y = \frac{-5}{3}$

3 $x = 4 \; and \; y = 27$

$x = -3 \; and \; y = -1$